

# RESONANT COMBUSTION IN ROCKETS

The variety of physical and chemical processes taking place during the release of energy in a rocket engine may create conditions for oscillations to build up to large amplitudes

by J. George Sotter and Gary A. Flandro

**S**olid-propellant rockets apparently were invented in China about 1,000 years ago. They did not attract much notice until the 13th century A.D., when they began to be used in warfare. From about the 14th century rockets were constantly competing with guns for superiority in battle. Not long after Francis Scott Key wrote of "the rockets' red glare" in 1814 the improved accuracy of guns seemed to have doomed rockets to a secondary role. Consequently the invention in 1895 of the first practical liquid-propellant rocket by Pedro Paulet, a Peruvian engineer, aroused little interest.

Three decades later the American physicist Robert H. Goddard launched a liquid-propellant rocket powered by liquid oxygen and gasoline. Goddard, whose work established the foundation of liquid-propellant technology, was at the time considered something of a crackpot by some of the American authorities who were following his efforts. His work, however, did not go unnoticed in Germany. By 1939 German rocket engineers were pressing to obtain funds for the development of a large liquid-propellant rocket, which later became known as *Vergeltungswaffe* ("vengeance weapon") 2.

The V-2 was to usher in the space age and pave the way for man's journey to the moon. In 1940, however, a British team developing a small solid-propellant rocket as an anti-aircraft weapon encountered unstable combustion—a phenomenon that has since then plagued nearly every major rocket-engine development program. The British rocket was being given a static ground test at the Woolwich Arsenal, and about halfway through the burning period the pressure in the rocket chamber suddenly rose to more than twice the expected level and then returned rapidly to normal. Such an

excursion would have destroyed a rocket engine of flight weight, but the experimenters were working with a test chamber of heavy construction, and it was not damaged.

To investigate the matter further they made firings in which they extinguished combustion when the high pressure appeared. Examining the surface of the partly burned solid fuel, they found that it was rippled. The orientation of the marks suggested that the gases inside the motor had been moving in an intense vortex.

Evidently, then, the instability was attributable to an unusual motion of the gas during combustion. The British workers damped this motion in a simple way: they attached a steel ruler to the forward end of the combustion chamber so that it projected into the flow of gas. Such "resonance rods," made in a variety of forms and materials, are still employed to control unstable combustion in solid-propellant engines. They do, however, have a number of drawbacks. As the size of rocket motors has increased, the rods have had to be bulkier. They interfere with ignition of the charge, they reduce the potential payload and sometimes they do not significantly reduce the instability. Although various other remedies are sometimes effective, unstable combustion in solid-propellant rockets remains a problem.

Liquid-propellant rocket engines did not at first suffer from unstable combustion. The problem materialized when the designers of the German V-2 rocket undertook to make a propellant injector that would be less difficult to fabricate than the one they were using. Their new injector worked well in a small test engine, but when they tried it in a full-sized model, the engine emitted an audible hum that was accompanied by severe vibration. The German workers

solved the problem by simply reverting to the original injector. Many modern liquid-propellant rockets, however, are significantly different from the V-2; they use more energetic propellants and have injectors that provide more efficient combustion. All these factors seem to encourage instability, so that instability also remains a problem in liquid-propellant rockets.

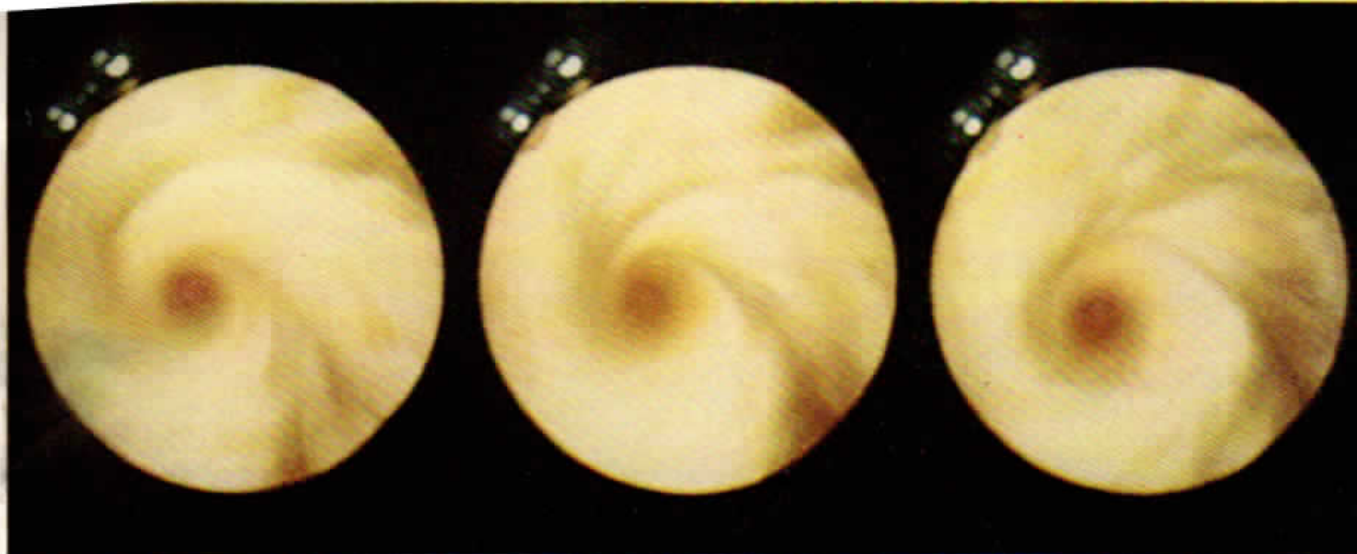
**B**y definition a rocket is a vehicle able to propel itself without aid from its environment. Automobile engines and gas-turbine jet engines use the surrounding air as an oxidizer to burn a fuel. The chemical rocket carries its own oxidizer as well as its fuel and thus can operate where there is no air.

Most of the rockets in service today are chemical rockets: they operate by releasing the chemical energy of the fuel and the oxidizer (referred to jointly as propellants). A few other kinds of rocket engine, such as nuclear engines and electrical ones, are in use or under development, but they will not concern us here.

A chemical rocket can have either solid or liquid propellants; in either case combustion converts the propellants into hot gases. The gases expand, creating high pressures that force them through a nozzle, where they become cooler as their thermal energy is converted to kinetic energy. They emerge from the nozzle at supersonic velocities, and the forward movement of the rocket is the recoil from their expulsion.

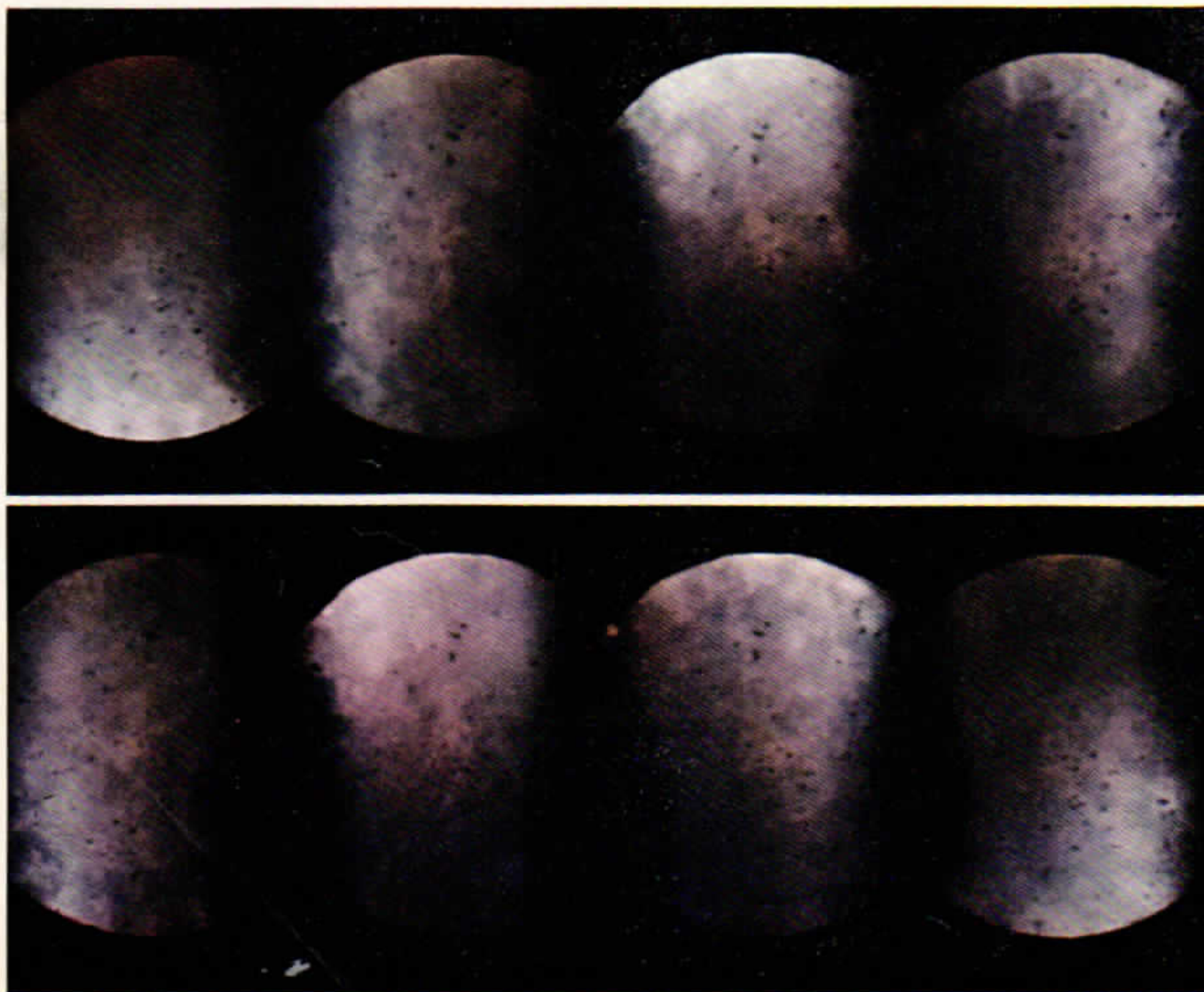
A solid propellant can be made by mixing a fuel and an oxidizer. (At the time of mixing one ingredient is in liquid form, but it subsequently hardens into a solid.) A typical mixture is ammonium perchlorate in a matrix of an organic fuel such as rubber. A homogeneous mixture of such self-oxidizing substances as nitrocellulose and nitroglycerin can





INTERIOR VIEW of a solid-propellant rocket shows a vortex resulting from combustion instability. Views, which are from a motion-picture film made by one of the authors (Sotter) and J. Swith-

enbank at the University of Sheffield, are through a window in the head end of the rocket and toward the nozzle from which gases escape. The nozzle is not visible because of smoke and luminosity.



DETONATION WAVE in a liquid-propellant rocket travels clockwise in this sequence of frames from a motion-picture film made by Richard M. Clayton of the Jet Propulsion Laboratory. The camera, mounted in a protective housing, was placed in the exhaust gases

just outside the nozzle. Hence the view is into the rocket engine toward the injector, which is a perforated metal plate at the head end of the motor. Through the injector fuel and oxidizer are admitted to the combustion chamber, where they are mixed and burned.

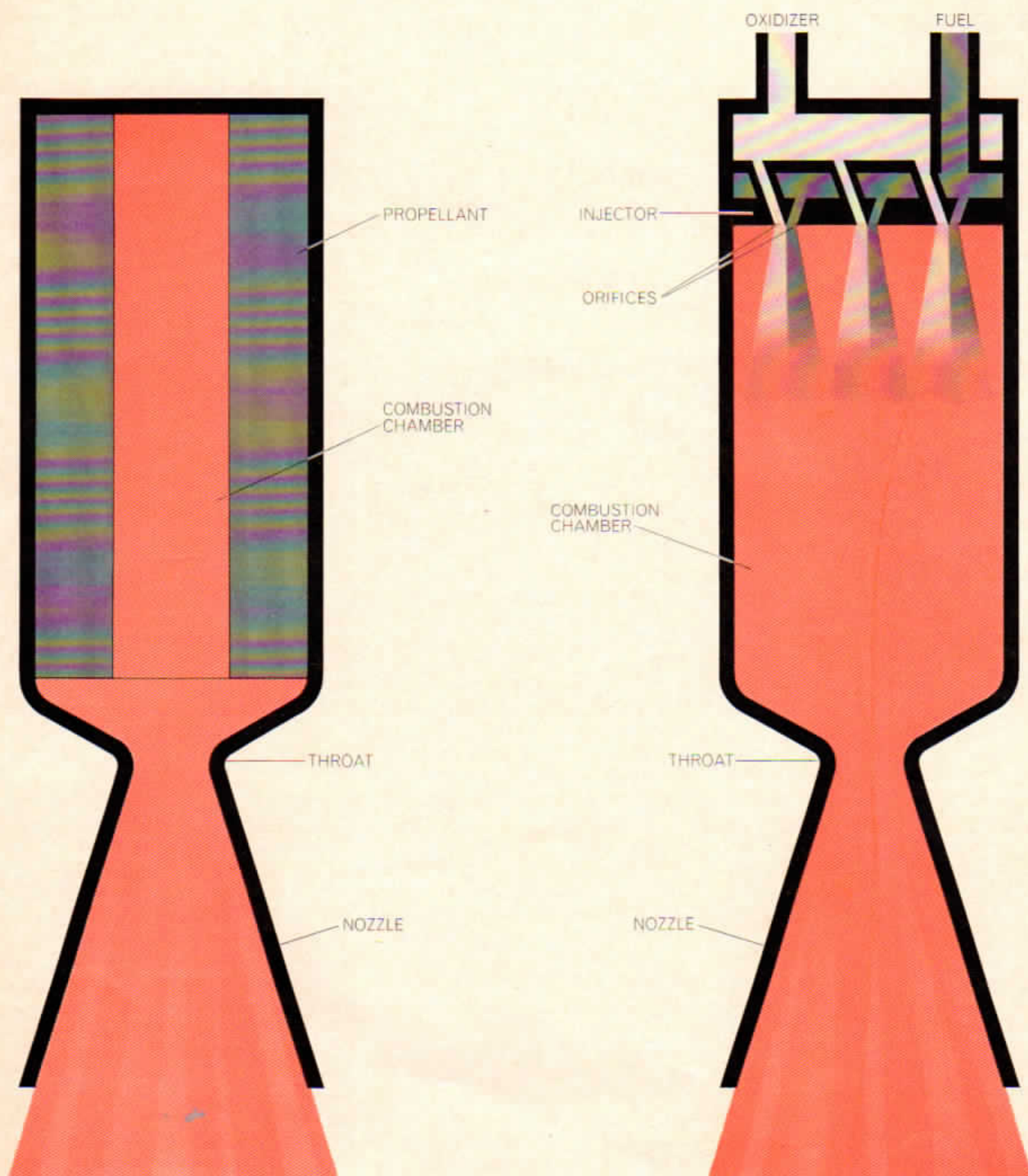
also be used. In either case the propellant is usually cast in such a way that a central column is left open. Combustion proceeds only on the surface of the exposed material.

In a liquid-propellant rocket engine the oxidizer and the fuel are fed sepa-

ately into the combustion chamber from storage tanks. They go through an injector that atomizes them and mixes them in the correct proportions so that they can be readily vaporized and burned. Among the oxidizers are liquid oxygen and liquid fluorine; the fuels in-

clude kerosene, alcohol and hydrazine.

A solid-propellant engine requires no pumps or pressurization, so that it is mechanically simpler than a liquid-propellant engine. On the other hand, a solid-propellant engine is not as controllable as one using liquids. Only re-



**CHEMICAL ROCKETS**, which are the kind predominantly in use at present, can use either a solid propellant (*left*) or liquid ones (*right*). A solid-propellant rocket can be the simplest of propulsive devices, consisting basically of a nozzle and a chamber containing

the charge. A liquid-propellant rocket burns a fuel and an oxidizer; they are stored elsewhere in the vehicle and fed into the combustion chamber through an injector, which is a perforated plate. A typical propellant combination is kerosene and liquid oxygen.



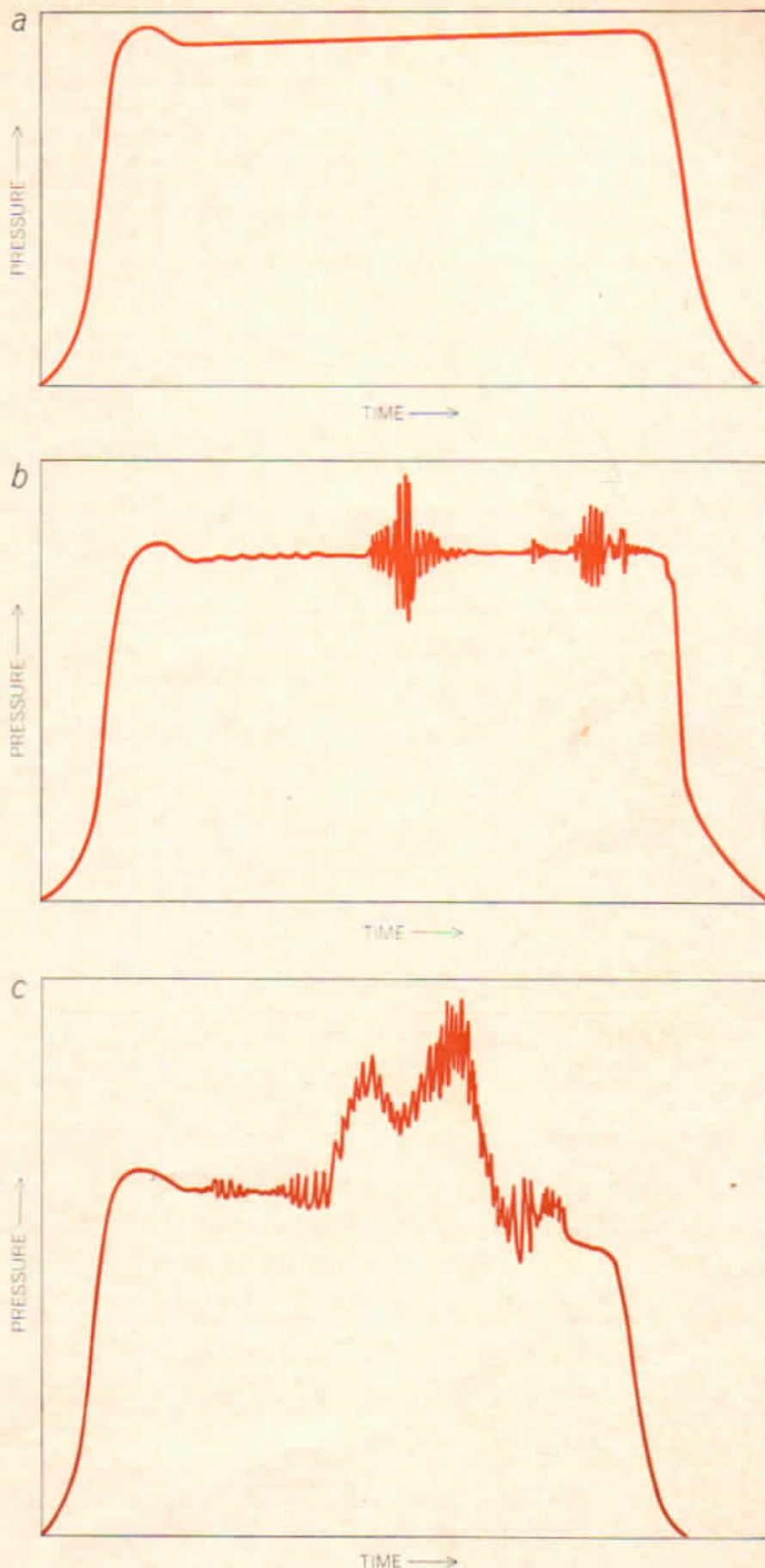
cently, for example, has it become possible to turn off a solid-propellant engine and restart it, whereas a liquid-propellant engine can be stopped and restarted readily. In general a liquid-propellant engine is capable of higher and more adjustable performance than a solid-propellant engine, which is why most of the engines for space vehicles are liquid-propellant ones.

Whether they are in solid or liquid form, the propellants can be regarded as a large reservoir of energy that can be tapped in various ways by the many physical and chemical processes that take place during operation, giving rise to the possibility of self-excited oscillations in the system. Oscillatory behavior is always possible when an energy source is coupled with an object or medium capable of vibratory motion, and when a feedback loop exists between the source of energy and the vibrating system. Perhaps the best-known example of this phenomenon is feedback in a public-address system. Under certain conditions a random sound is picked up by the microphone and fed through the amplifier to the loudspeaker, from which it is again detected by the microphone. The impulse gains strength as it passes through the amplifier (a source of energy), and as the process is successively repeated the sound may build up until it becomes a piercing scream. Damping mechanisms (which are related to room acoustics, amplifier characteristics and other conditions) cause a leveling off of the volume when the gains and losses come into balance.

A similar process takes place in the unstable rocket motor. The analogue of the electronic amplifier is the combustion region in the combustion chamber. Under certain conditions this region can amplify sound waves, and if an appropriate feedback is provided by the boundaries of the chamber, the waves can grow to an intensity dependent on the damping processes that are acting.

Oscillatory behavior in rocket engines takes many forms, some of which are still poorly understood from a theoretical point of view. One form that appears in liquid-propellant engines is a low-frequency vibration, with a chugging sound, that is caused by coupling between the propellant-feed system and the combustion process. The two are coupled because the propellant's rate of flow is sensitive to the pressure in the combustion chamber.

As the pressure in the chamber increases, perhaps at first because of a



**PRESSURE RECORDS** characterize various types of combustion in rockets. Normal steady burning (a) produces a curve with a smooth profile. Oscillatory burning (b) shows various frequencies, sometimes with varying amplitudes. Radical departures of pressure from normal (c) are called irregular burning; they are apparently caused by oscillatory burning.



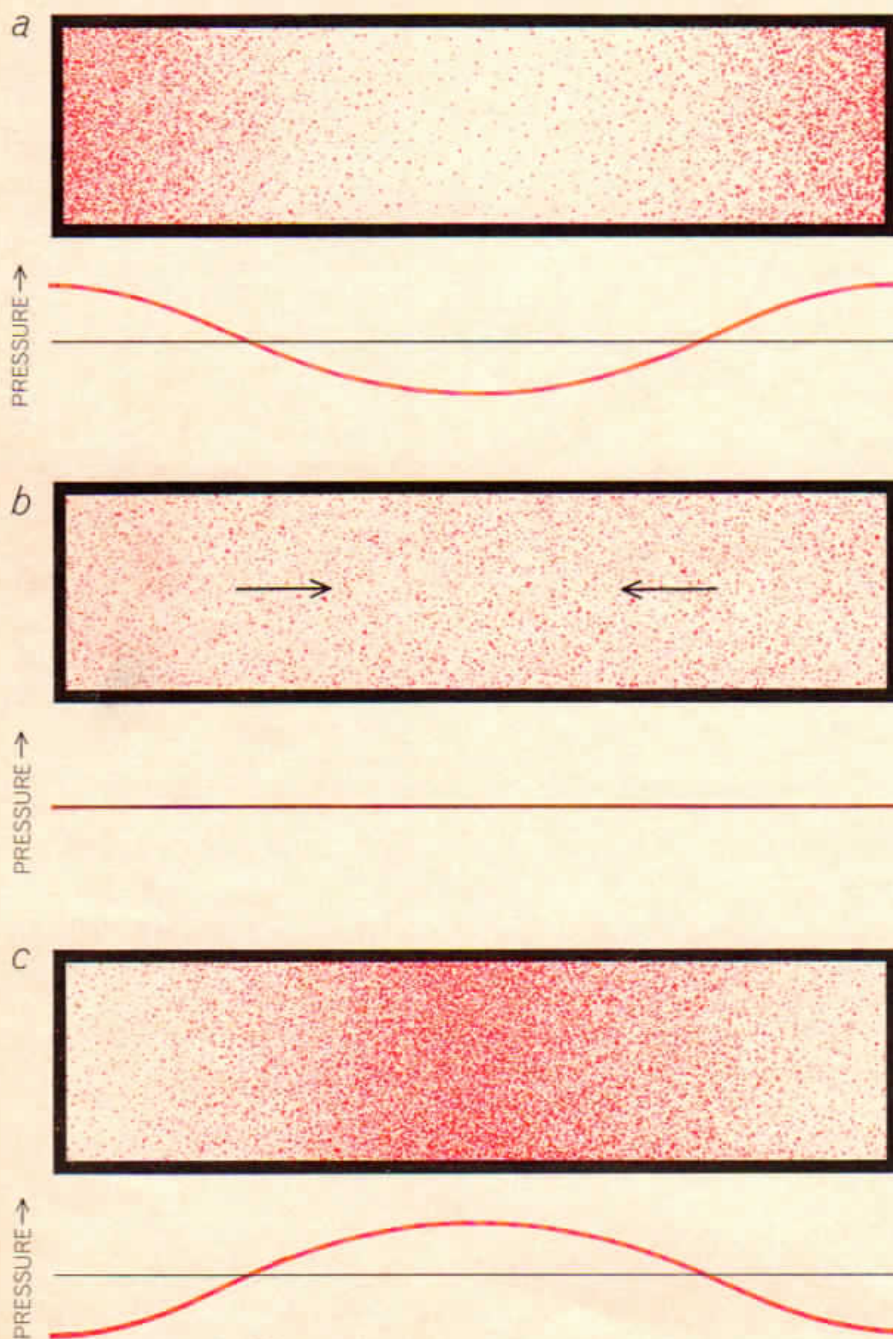
random disturbance, the rate of flow of the propellants through the injector tends to decrease. The slowing rate of flow brings about a decrease in the pressure in the combustion chamber, which encourages an increase in the rates of flow of the propellants. This type of instability could not develop if the system had appreciably different characteristic times for the rise and fall of pressure in the chamber and for increases and

decreases in the rates of flow of the propellants. The problem can be solved by tailoring the length of the lines that feed in the propellant, by changing the design of the injector and by other steps that have the same effect: making the characteristic times so different that resonance cannot develop.

A more destructive form of instability, found in both liquid-propellant and solid-propellant rockets, is called acous-

tic instability. It is basically characterized by high-frequency variations in the pressure of the combustion chamber. The resulting scream from the engine can often be clearly heard.

Acoustic instability can begin with small disturbances in the gas in the combustion chamber. These perturbations are amplified by the combustion process and by the flow of gas through the motor. The sound waves bounce off the boundaries of the chamber and so are fed back into the system. In solid-propellant engines the phenomenon leads to sharp rises in pressure; in liquid-propellant rockets it is the oscillations themselves that can produce severe damage by causing large vibrational accelerations and by transferring excess heat to the walls of the engine.



**ACOUSTIC OSCILLATION** of the longitudinal type is depicted schematically for a closed tube. The acoustic oscillation, or sound wave, is assumed to be present as one begins observing the process (a) at a time when all the particles, or groups of molecules, of gas are at rest and distributed as shown. The graph below represents the distribution of pressure. Soon (b) the particles are uniformly dispersed but most of them have a velocity toward the center, as indicated by the arrows. Later (c) the particles bunch in the center, building up a pressure that brings them to rest and then forces them back toward the ends.

The effort to eliminate oscillatory combustion in rockets has been largely based on trial and error, which is unavoidably costly and time-consuming. The difficulties would be much eased if there were a theory that would predict the occurrence of high-amplitude waves in a motor of given design and would allow the investigation of possible ways of eliminating them. The first attempt at such a theory was published in 1942 by the Russian physicist Ya. B. Zel'dovich, but attempts to use his analysis to predict the stability of real engines were unsuccessful. Notwithstanding intensive efforts by many investigators in the years since then, a completely satisfactory theory does not yet exist. The reasons for this state of affairs will perhaps be made clearer by a brief outline of the physical processes involved in oscillatory combustion.

The discipline of acoustics has furnished a frame of reference for much of the theoretical work on instability in rocket combustion. A particularly helpful mathematical formulation has been the acoustic wave equation, which deals with the various ways gases can oscillate. In a closed circular cylinder, which is an approximation of the geometry of certain rocket engines, three "pure" modes of oscillation are possible: longitudinal, radial and tangential. The longitudinal and radial types, as their names imply, involve motions only along the length or the radius of the cylinder. The tangential type involves motions that have both tangential and radial components but that at the wall of the cylinder are only tangential. The three modes can of course be present simultaneously and in various combinations. In addition various modes with inherently com-



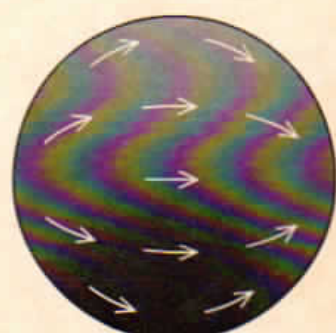
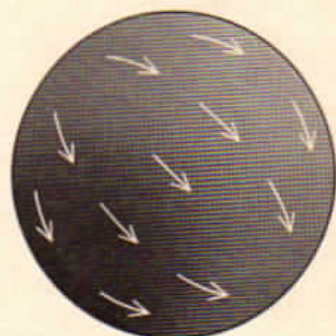
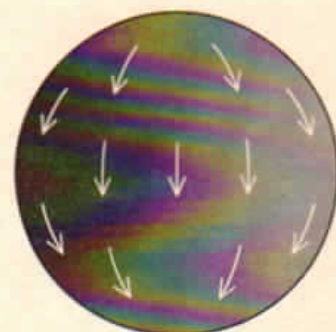
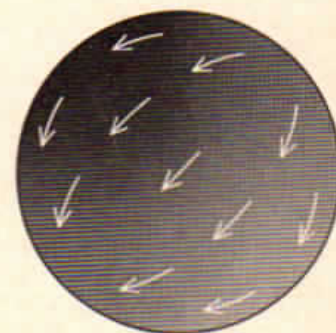
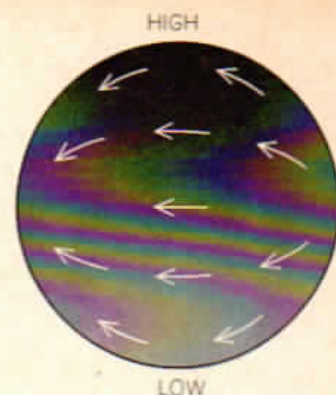
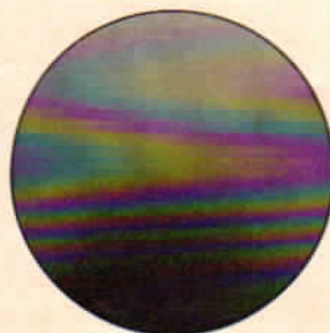
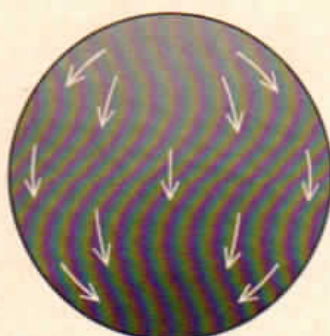
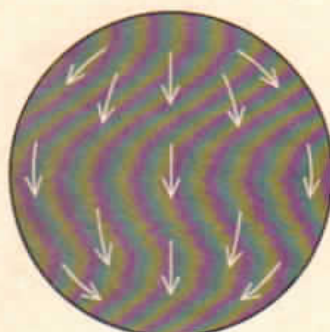
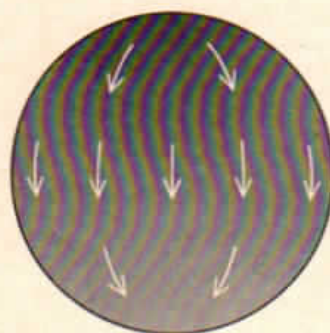
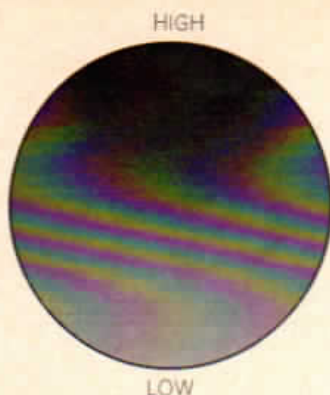
bined longitudinal, radial and/or tangential motion are possible.

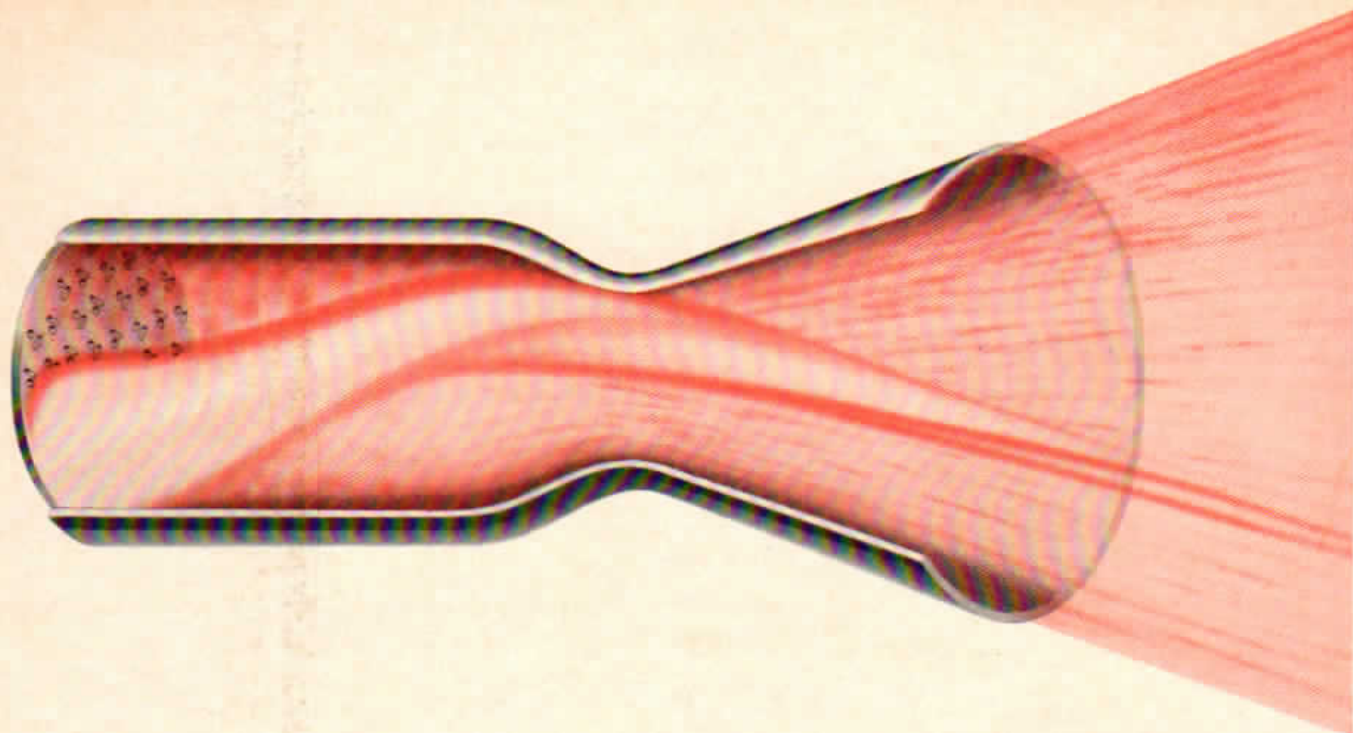
The acoustic wave equation is rather easily solved for vessels of simple geometry, chiefly because it is a linear equation. This important property is achieved at the expense of assuming that the variations in pressure (and the associated variations in gas velocity) are infinitesimal at all times. In classical acoustics such assumptions serve well: ordinary conversation produces pressure excursions of about a millionth of a pound per square inch. In a rocket motor, however, pressure pulses a billion times stronger than such changes have been observed. Thus, as one might expect, there are fundamental differences between the acoustics of a rocket motor and those of a closed resonant chamber. The strong oscillations that can occur in a rocket motor cannot be described by the classical wave equation.

Beyond the high-amplitude effects the acoustics of rocket motors are complicated by the presence of burning solids or liquids; they can add acoustic energy to the system at enormous rates. Another direct source of acoustic energy is the high-speed flow of gases through the combustion chamber. Therefore one must derive more elaborate equations, starting with the laws of conservation of mass, energy and momentum.

Sound waves in a gas are propagated by molecular collisions, and the speed of sound is of the same order of magnitude as the average molecular speed. In acoustics, however, it is not usually necessary to deal with the statistics of astronomical numbers of molecules. The gas can be thought of as being composed of "particles," or groups of molecules, each of which includes a large number of molecules. Taken together, the particles form a continuum having a definite pressure, density, temperature and velocity at each point. For the gas phase these four variables are related by the laws of mass, energy and momentum conservation together with an equation of state (such as the ideal-gas law). Similar rela-

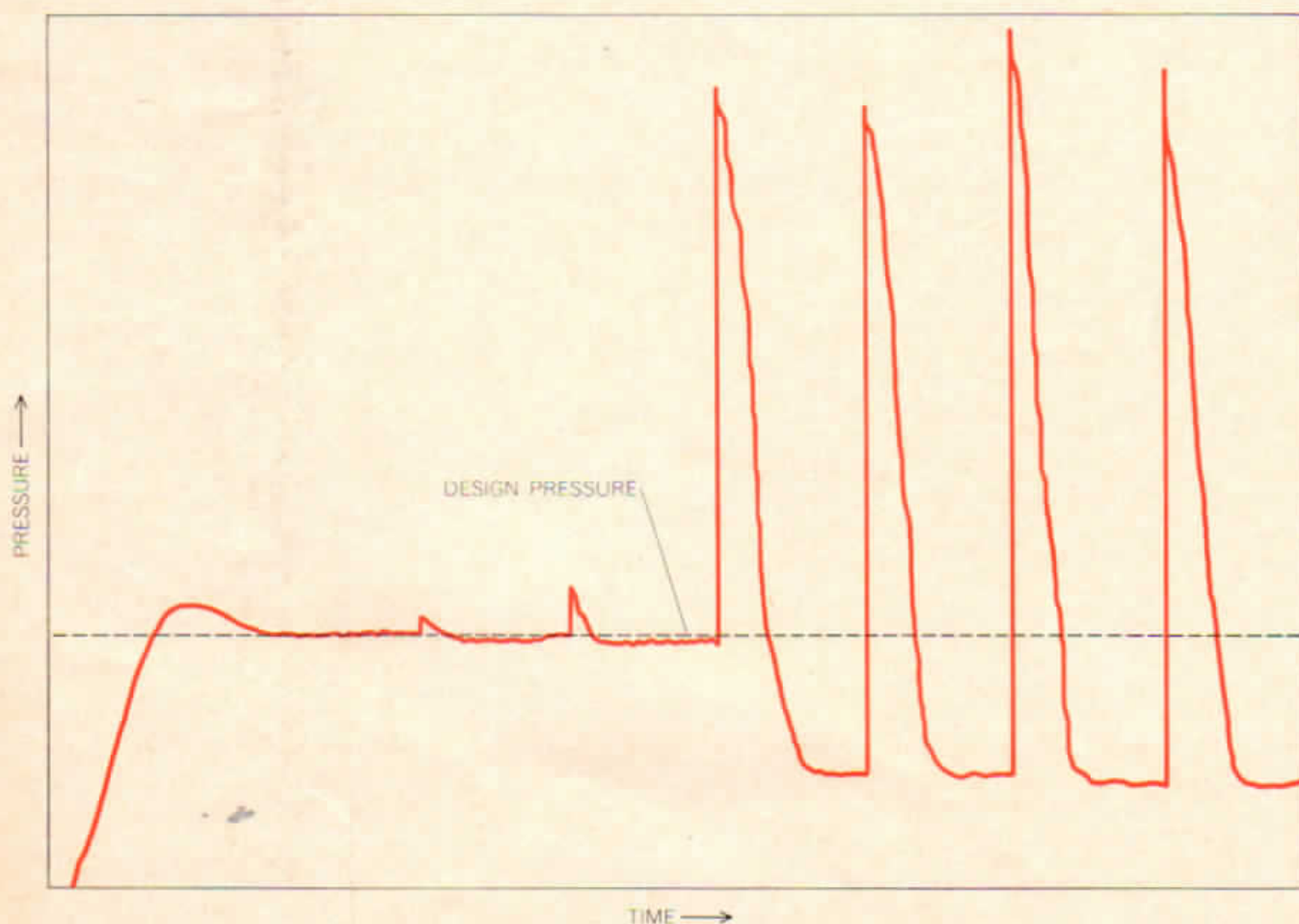
**TANGENTIAL FORM** of instability appears in two ways, the standing wave (*left*) and the traveling wave (*right*). In each case the changes of pressure from high (*dark*) to low (*light*) and of velocity (*arrows*) are depicted for a half-cycle. The standing mode is analogous to the sloshing that would be produced if a coffee cup were tilted back and forth; the traveling mode resembles what happens if the cup is moved in a circle.





ROTATING WAVE resembles a detonation. Such a wave is believed to have this appearance as it travels around the combustion chamber of a liquid-propellant rocket. In the bright zone near the

face of the injector (*left*) the wave is steep-fronted and extremely strong. Movement is clockwise as viewed from the nozzle end of the rocket. Similar action appears in photographs at bottom of page 94.



RECORD OF PRESSURE produced by a rotating wave such as the one depicted in the top illustration on this page differs markedly from the sinusoidal form produced by an acoustic distur-

ance. The wave may develop from an acoustic disturbance or it may appear suddenly, as in the condition represented here. Ratios of pressure across such a wave often are more than 20 to one.



tions apply to the solid or liquid propellants.

The two sets of equations are strongly coupled, since the behavior of the gases affects the behavior of the condensed phases and vice versa. Heat from the combustion gases, which are typically at 5,000 degrees Fahrenheit, causes evaporation of the propellants, which apart from a thin heated surface layer may be at or below normal room temperature. Rapidly moving gases increase heat-transfer rates and can erode solids or shatter liquid drops, so that vaporization rates may momentarily be greatly increased. After a short time the additional vapors release their chemical energy through combustion. If only a small percentage of this energy is diverted to drive oscillations, pressure waves of enormous amplitude may be created.

In studying unstable combustion in rockets, one is interested only in the part of the engine upstream of the engine's throat. In the diverging section downstream of the throat the flow is supersonic; any small pressure disturbances (which travel at the speed of sound) will be swept downstream so fast they cannot reach the combustion chamber. For this reason the chamber is considered to be bounded on the downstream end by the gaseous "sonic surface" that exists at the throat.

The tangential modes of oscillation are of special interest because they are apparently the most destructive types of instability. The first tangential mode is best described as a swashing of gases around the combustion chamber; the phenomenon resembles the swashing of a liquid in a cylindrical container. At the chamber walls the velocity nodes, which are the points where the gas velocity is zero, may remain stationary. In this case the motion is termed a standing or sloshing wave. Alternatively, the nodes may move around the walls of the chamber. Here the motion is referred to as the traveling or swashing form of tangential instability.

The traveling form, when it is present at high amplitudes, produces a number of noteworthy effects. One of them is the development of detonation waves, which can be loosely defined as shock waves driven by combustion [see top illustration on opposite page]. A shock wave (a familiar form of which is the sonic boom) is an extremely steep-fronted disturbance that sometimes develops from a sinusoidal, or acoustic, type of disturbance. Because gases are heated by compression, the high-pressure part of the

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sine wave (where the speed of sound, which is proportional to the square root of the absolute temperature, may be appreciably higher than average) can travel faster than the portions that are at lower pressures. As a result the waves steepen, somewhat in the way that ocean waves steepen into breakers on a beach.

Once a shock wave has formed in a liquid-propellant motor the large gradients of temperature, pressure and velocity can promote drastic changes in rates of combustion. At the walls of the chamber such a wave can increase heat transfer to rates higher than those obtained from an oxyacetylene torch. The chamber can be cut apart in less than a second.

In a solid-propellant motor the wall is protected by the propellant. Here the most striking effect of the traveling tangential wave is the sharp increase it produces in the mean operating pressure of the engine. The effect can lead to the explosion of the rocket engine, and at the very least it impairs performance.

This phenomenon is called irregu-

lar combustion. Motion pictures made through the forward end of a solid-propellant engine suggest that the irregularity is the result of the formation of a vortex—a tornado-like motion of the gases [see top illustration on page 94]. The vortex is created by the traveling tangential wave. Inside the vortex is a low-pressure core like the eye of a tornado, and the combined effect of the low-pressure core and the swirling can severely hamper the flow of gases through the nozzle. In such a case gases may be generated inside the motor faster than they can escape through the nozzle; the result is a rise of pressure in the motor. With some propellants the rate of burning is increased by the rise of pressure and by the high velocity in the vortex, so that the pressure in the chamber goes up even more. These are the conditions that can cause the motor to explode.

When the vortex is less severe, its most obvious effect may be to generate a torque around the longitudinal axis of the rocket. Torque is difficult to measure in ground tests, and the measure-

ment is seldom attempted. Thus the existence of both the vortex and the torque has sometimes been revealed only when a rocket was tested in flight.

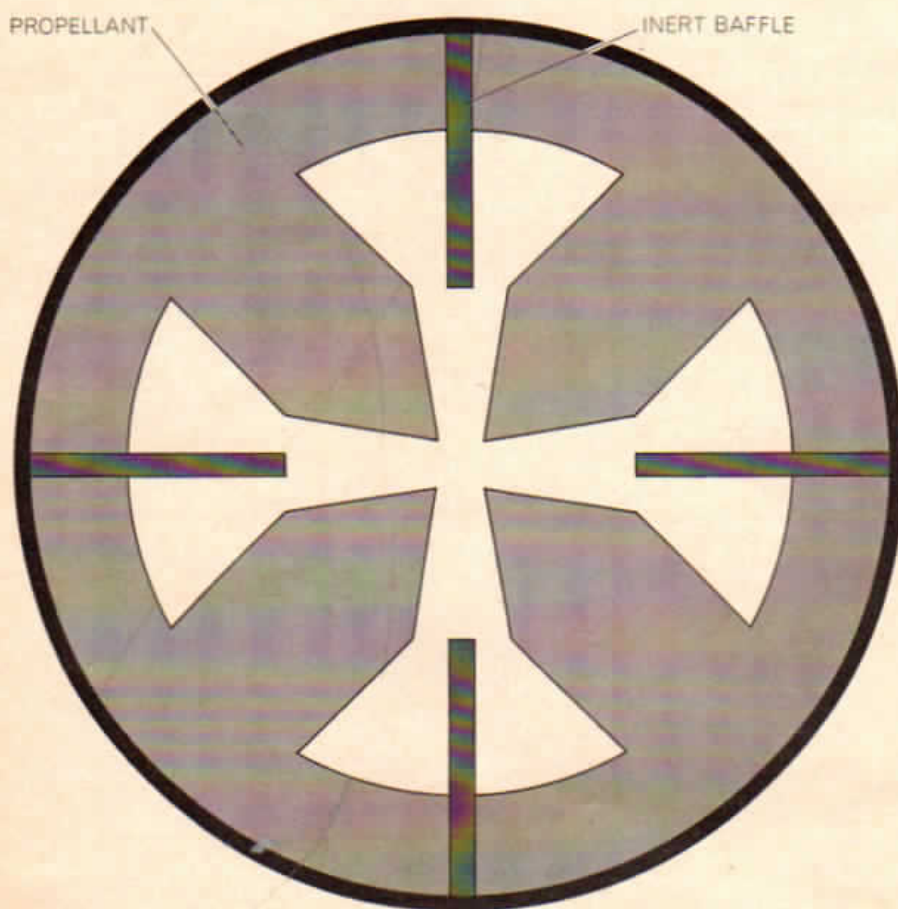
It was the symptoms of vortex formation that the British group discovered at the Woolwich Arsenal in 1940. For reasons that are difficult to explain, the existence of vortices and the possible side effects seem to have been largely ignored in solid-propellant research since that time. Only in recent years has the significance of the phenomenon been understood.

The rocket designer faced with the problem of suppressing instability in an engine usually attempts to increase the influence of damping effects. A technique sometimes employed with liquid-propellant rockets is to put perforated, sound-absorbing inner walls in the combustion chamber. This solution will not do for solid-propellant motors, in which the propellant lines the walls. It has been found helpful to add metallic particles to a solid propellant. This produces particles of solid or liquid metal oxide in the gases. The oxide particles dissipate acoustic energy. The effect is quite similar to the attenuation of sound in fog, and in many solid-propellant engines it eliminates combustion instability.

Where these various techniques are inadequate or unacceptable there are other alternatives. For solid-propellant rockets it is possible to change the composition or the geometry of the propellant. With liquid-propellant rockets one can change the ratio of fuel to oxidizer or the design of the injector.

These techniques may entail a loss in combustion efficiency. It might seem that, since rocket engines are highly efficient in using the available energy, such a loss would be a small price to pay for stable combustion. With a vehicle the size of the U.S. rocket *Saturn V*, however, a loss of 3 percent in combustion efficiency would require the use of another tank car full of propellants to accomplish the same mission. The result could easily be a drastic reduction in the payload of the vehicle.

A recurring difficulty for designers is that test results in many unstable engines have been difficult to reproduce. With liquid-propellant rockets the trouble lies in the hydraulics of the injection system. A typical injector consists basically of a metal plate that is about 1/4 inch thick and has holes drilled in it to admit propellant to the combustion chamber. Tiny jagged edges on the upstream end of the holes (often in



DAMPING DEVICE in a solid-propellant rocket can be arrangement of baffles between segments of the propellant. A typical geometrical arrangement of propellant is shown in this cross section; the shape provides a large amount of surface for burning. The baffles are designed to break up combustion instabilities before they reach serious proportions.



conjunction with strong cross-velocities in the flow of propellant) make the flow of liquids through the individual holes rather unpredictable. Moreover, slight variations from engine to engine in the machining of injector holes make it difficult to be confident that two otherwise identical engines, from the same assembly line, will have identical stability.

Similar problems are met in solid-propellant motors. Changes in propellant temperature, for instance, can have a drastic effect on stability. Thus a motor that performs perfectly on a cold day may exhibit violent instability on a hot day.

Perhaps the best hope for progress in preventing instability in solid-propellant combustion is a laboratory device called the T burner: a T-shaped solid-propellant motor that is used to measure the amplification tendencies of various propellants. Both theory and experiment have shown that most propellants are capable of amplifying oscillations over a wide range of frequencies. The T burner provides a controlled environment in which the damping mechanisms and the geometry of the combustion chamber are relatively simple. The T burner, with its capability for measuring amplification, adds an important kind of information to the investigator's supply of facts by making it possible for him to characterize the least understood part of the unstable system: the combustion process itself. One can hope that the knowledge of a propellant's amplifying characteristics obtained inexpensively from the T burner will eventually be applied reliably to the prediction of what will happen in a large rocket engine.

Since instability of combustion is sensitive to small changes in engine geometry and operating conditions, a particular engine must be subjected to a large number of firings before its designers can say confidently that it is free from instability. With a large engine such testing can account for a substantial part of development costs. Herein lies the importance of devising reliable theories of instability and inexpensive tests of a propellant's acoustical characteristics. Until instability of combustion is understood well enough so that it can be eliminated while an engine is in the design stage, rockets must continue to be intensively tested for stability—particularly when the lives of astronauts will eventually depend on safe, reliable operation of the engine.

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